A Bound Strengthening Method for Optimal Transmission Switching in Power Systems

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***Abstract*—This paper studies the optimal transmission switching (OTS) problem for power systems, where certain lines are fixed (uncontrollable) and the remaining ones are controllable via on/off switches. The goal is to identify a topology of the power grid that minimizes the cost of the system operation while satisfying the physical and operational constraints. Most of the existing methods for the problem are based on first converting the OTS into a mixed-integer linear program (MILP) or mixed-integer quadratic program (MIQP), and then iteratively solving a series of its convex relaxations. The performance of these methods depends heavily on the strength of the MILP or MIQP formulations. In this paper, it is shown that finding the strongest variable upper and lower bounds to be used in an MILP or MIQP formulation of the OTS based on the big-M or McCormick inequalities is NP-hard. Furthermore, it is proven that unless $P = NP$, there is no constant-factor approximation algorithm for constructing these variable bounds. Despite the inherent difficulty of obtaining the strongest bounds in general, a simple bound strengthening method is presented to strengthen the convex relaxation of the problem when there exists a connected spanning subnetwork of the system with fixed lines. With the proposed bound strengthening method, remarkable improvements in the runtime of the mixed-integer solvers and the optimality gaps of the solutions are achieved for medium- and large-scale real-world systems.**

***Index Terms*—Optimal transmission switching, mixed-integer optimization, algorithms, global optimization, economic dispatch**

I. Introduction

In power systems, transmission lines have traditionally been considered uncontrollable infrastructure devices, except in the case of an outage or maintenance. However, due to the pressing needs to boost the sustainability, reliability and efficiency, power system directors call on leveraging the flexibility in the topology of the grid and co-optimizing the production and topology to improve the dispatch. In the last few years, Federal Energy Regulatory Commission (FERC) has held an annual conference on "Increasing Market and Planning Efficiency through Improved Software" [1] to encourage research on the development of efficient software for enhancing the efficiency of the power systems via optimizing the flexible assets (e.g., transmission switches) in the system. Furthermore, The Energy Policy Act of 2005 explicitly addresses the "difficulties of siting major new transmission facilities" and calls for the utilization of better transmission technologies [2].

Unlike in the classical network flows, removing a line from a power network may improve the efficiency of the network due to physical laws. This phenomenon has been observed and harnessed to improve the power system performance by many authors. The notion of *optimally switching the lines of a transmission network* was introduced by O'Neill *et al.* [3]. Later on, it has been shown in a series of papers that the incorporation of controllable transmission switches in a grid could relieve network congestions [4], serve as a corrective action for voltage violation [5]–[7], reduce system loss [8], [9] and operational costs [10], improve the reliability of the system [11], [12] and enhance the economic efficiency of power markets [13]. We refer the reader to Hedman *et al.* [14] for a survey on the benefits of transmission switching in power systems. However, the identification of an optimal topology, namely optimal transmission switching (OTS) problem, is a non-convex combinatorial optimization problem that is proven to be NP-hard [15]. Therefore, brute-force search algorithms for finding an optimal topology are often inefficient. Most of the existing methods are based on heuristics and iterative relaxations of the problem. These methods include, but are not restricted to, Benders decomposition [10], [12], branch-and-bound and cutting-plane methods [16], [17], genetic algorithms [7], and line ranking [18], [19]. Recently, another line of work has been devoted to strong convexification techniques in solving mixed-integer problems for power systems [20]–[22].

In this work, the power flow equations are modeled using the well-known DC approximation, which is the backbone of the operation of power systems. Despite its shortcomings for the OTS in some cases [23], the DC approximation is often considered very useful for increasing the reliability, performance, and market efficiency of power systems [14]. The OTS consists of disjunctive constraints that are bilinear and nonconvex in the original formulation. However, all of these constraints can be written in a linear form using the so-called big-M or McCormick inequalities [24], [25]. This formulation of OTS is referred to as the *linearized OTS* in the sequel. A natural question arising in constructing the OTS formulation is: how can one find optimal values for the parameters of the big-M or McCormick inequalities?

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An optimal choice for these parameters is important for two reasons: 1) they would result in stronger convex relaxations of the problem, and hence, fewer iterations in branch-and-bound or cutting-plane methods, and 2) a conservative choice of these parameters would cause numerical and convergence issues [26]. Hedman *et al.* [11] point out that finding the optimal values for the parameters of the linearized OTS may be cumbersome, and, therefore, they impose restrictive constraints on the absolute angles of voltages at different buses at the expense of shrinking the feasible region. Other studies [3], [10], [16] have also used similar restrictive approaches to solve the linearized OTS.

In this work, it is proven that finding the optimal values for the parameters of the MILP or MIQP formulations of the OTS using either big-M or McCormick inequalities is NP-hard. Moreover, it is shown that there does not exist any polynomial-time algorithm to approximate these parameters within any constant factor, unless $P = NP$. This new result adds a new dimension to the difficulty of the OTS; not only is solving the OTS as a mixed-integer nonlinear program difficult, but finding a good linearized reformulation of this problem is NP-hard as well. In order to maintain the reliability and security of the system, often a set of transmission lines are considered as fixed and the flexibility in the network topology is limited to the remaining lines. An implicit requirement is that the network should always remain connected in order to prevent islanding. One way to circumvent the islanding issue in the optimal transmission switching problem is to include additional security constraints in order to keep the underlying network connected at every feasible solution [27], [28]. However, this new set of constraints would lead to the over-complication of an already difficult problem. Therefore, in practice, many energy corporations, such as PJM and Exelon, consider only a selected subset of transmission lines as flexible assets in their network [29], [30].

In this paper, it is proven that the OTS with a fixed connected spanning subnetwork is still NP-hard but one can find non-conservative values for the parameters of the big-M or McCormick inequalities in the linearized OTS without shrinking the feasible region or sacrificing the optimality of the obtained solution. In particular, a simple bound strengthening method is presented to strengthen the linearized formulation of the OTS. This method can be integrated as a preprocessing step into any numerical solver for the OTS. Despite its simplicity, it is shown through extensive case studies on the IEEE 118-bus system and different Polish networks that the incorporation of the proposed bound strengthening method leads to substantial speedup in the runtime of the solver. Furthermore, it is shown that while including additional constraints on the absolute values of the angles at different buses can improve the runtime of the solver, it may steer away from the optimality; this conservative approach can increase the operation cost by 7% for Polish networks.

II. Problem Formulation

Consider a power network with n_b buses, n_g generators, and n_l lines. This network can be represented by a directed graph, denoted by $G(\mathcal{B},\mathcal{L})$, where $\mathcal{B}$ is the set of buses indexed from 1 to n_b and $\mathcal{L}$ is the set of lines whose directions are chosen arbitrarily and indexed as (i,j) to represent a connection between buses i and j. Denote $\mathcal{G} = \{1, 2, ..., n_g\}$ as the set of generators in the system. Furthermore, let $\mathcal{N}_g(i)$ be the indices of generators that are connected to bus i. Note that $\mathcal{N}_g(i)$ may be empty for a bus i. The variable p_i corresponds to the active-power production of generator $i \in \mathcal{G}$ and the variable θ_i is the voltage angle at bus $i \in \mathcal{B}$. For every $(i,j) \in \mathcal{L}$, the variable f_{ij} denotes the active flow from bus i to bus j. Consider the set of lines $\mathcal{S} \subseteq \mathcal{L}$ that are equipped with on/off switches and define the decision variable x_{ij} for every $(i,j) \in \mathcal{S}$ as the status of the line (i,j). Let n_s denote the cardinality of this set. We refer to the lines belonging to $\mathcal{S}$ as *flexible* lines and the remaining lines as *fixed* lines. Notice that the decision variables p_i, θ_i, and f_{ij} are continuous, whereas x_{ij} is binary. For simplicity of notation, define the variable vectors

$$
\begin{aligned}
&\mathbf{p} \triangleq [p_1, p_2, ..., p_{n_g}]^\top,\\
&\Theta \triangleq [\theta_1, \theta_2, ..., \theta_{n_b}]^\top,\\
&\mathbf{f} \triangleq [f_{i_1 j_1}, f_{i_2 j_2}, ..., f_{i_{n_l} j_{n_l}}]^\top,\\
&\mathbf{x} \triangleq [x_{i_1 j_1}, x_{i_2 j_2}, ..., x_{i_{n_s} j_{n_s}}]^\top,
\end{aligned} \tag{1}
$$

where the lines in $\mathcal{L}$ are labeled as $(i_1, j_1), ..., (i_{n_l}, j_{n_l})$ such that the first n_s lines denote the members of $\mathcal{S}$. The objective function of the OTS is defined as $\sum_{i\in\mathcal{G}} g_i(p_i)$, where $g_i(p_i)$ takes the quadratic form $g_i(p_i) = a_i \times p_i^2 + b_i \times p_i$ with $a_i \neq 0$ or the linear form $g_i(p_i) = b_i \times p_i$, for some numbers $a_i, b_i \geq 0$. In this paper, we consider both quadratic and linear objective functions, which may correspond to system loss and operational cost of generators. Every in-operation power system must satisfy operational constraints arising from physical and security limitations. The physical limitations include the unit and line capacities. Furthermore, the power system must satisfy the power balance equations. On the security side, there may be a cardinality constraint on the maximum number of flexible lines that can be switched off in order to avoid endangering the reliable operation of the system. Let the vector $\mathbf{d} = [d_1, d_2, ..., d_{n_b}]^\top$ collect the set of demands at all buses. Moreover, define $p_i^{\min}$ and $p_i^{\max}$ as the lower and upper bounds on the production level of generator i, and $f_{ij}^{\max}$ as the capacity of line $(i,j) \in \mathcal{L}$. Each line $(i,j) \in \mathcal{L}$ is associated with susceptance B_{ij}.

Using the above notations, the OTS is formulated as the following mixed-integer nonlinear problem:

$$\underset{\mathbf{f},\mathbf{x},\mathbf{\Theta},\mathbf{p}}{\text{minimize}} \quad \sum_{i\in\mathcal{G}} g_i(p_i) \tag{2a}$$

$$\text{s.t.} \qquad x_{ij} \in \{0,1\}, \qquad \forall (i,j)\in\mathcal{S} \tag{2b}$$

$$p_k^{\min} \le p_k \le p_k^{\max}, \qquad \forall k\in\mathcal{G} \tag{2c}$$

$$-f_{ij}^{\max} x_{ij} \le f_{ij} \le f_{ij}^{\max} x_{ij}, \quad \forall (i,j)\in\mathcal{S} \tag{2d}$$

$$-f_{ij}^{\max} \le f_{ij} \le f_{ij}^{\max}, \qquad \forall (i,j)\in\mathcal{L}\backslash\mathcal{S} \tag{2e}$$

$$B_{ij}(\theta_i-\theta_j)x_{ij} = f_{ij}, \qquad \forall (i,j)\in\mathcal{S} \tag{2f}$$

$$B_{ij}(\theta_i-\theta_j) = f_{ij}, \qquad \forall (i,j)\in\mathcal{L}\backslash\mathcal{S} \tag{2g}$$

$$\sum_{k\in\mathcal{N}_g(i)} p_k - d_i = \sum_{(i,j)\in\mathcal{L}} f_{ij} - \sum_{(j,i)\in\mathcal{L}} f_{ji}, \ \forall i\in\mathcal{B} \tag{2h}$$

$$\sum_{(i,j)\in\mathcal{S}} x_{ij} \ge r, \tag{2i}$$

where

- (2b) states that the status of each flexible line must be binary;
- (2c) imposes lower and upper bounds on the production level of generating units;
- (2d) and (2e) state that the flow over a flexible or fixed line must be within the line capacities when its switch is on, and it should be zero otherwise;
- (2f) and (2g) relate the flow over each line to the voltage angles of the two endpoints of the line if it is in service, and it sets the flow to zero otherwise;
- (2h) requires that the power balance equation be satisfied at every bus;
- (2i) states that at least r flexible lines must be switched on.

The reasoning behind incorporating the minimum cardinality constraint (2i) in the OTS is twofold:

- A small number of switching options is often essential to guarantee the practicality of different methods and a cardinality constraint on the maximum number of switchable lines is imposed to ensure this assumption [18], [19], [31].
- This lower bound is also used to guarantee the reliability of the network, especially when the switching is used as a post-contingency recourse action in the real-time operation of power systems [11], [31].

Define $\mathcal{F}$ as the feasible region of (2), i.e., the set of $\{\mathbf{f},\mathbf{x},\mathbf{\Theta},\mathbf{p}\}$ satisfying (2b)- (2i).

Due to space restrictions, we consider only one time slot of the system operation. However, the techniques developed in this paper can also be used for the OTS over multiple time slots with coupling constraints, such as ramping limits on the productions of the generators. As another generalization, one can consider a combined unit commitment and optimal transmission switching problem [10], [32], [33]. In this paper, the term "optimal solution" refers to a globally optimal solution rather than a locally optimal solution.

III. Linearization of OTS

The aforementioned formulation of the OTS belongs to the class of mixed-integer nonlinear programs. The nonlinearity of this optimization problem is, in part, caused by the multiplication of the binary variable x_{ij} and the continuous variables θ_i and θ_j in (2f). However, since this nonlinear constraint has a disjunctive nature, one can use the big-M or McCormick reformulation technique to formulate it in a linear way. First, we consider the big-M method, and then show that the same result holds for the McCormick reformulation scheme in the OTS. One can re-write (2f) for each flexible line (i,j) in the form

$$B_{ij}(\theta_i-\theta_j)-M_{ij}(1-x_{ij}) \le f_{ij} \le B_{ij}(\theta_i-\theta_j)+M_{ij}(1-x_{ij}) \tag{3}$$

for a *large enough* scalar M_{ij}, which results in the *linearized OTS formulation.* The above inequality implies that if x_{ij} equals 1, then the line is in service and needs to satisfy the physical constraint $f_{ij} = B_{ij}(\theta_i-\theta_j)$. On the other hand, if x_{ij} equals 0, then (3) (and hence (2f)) is redundant as it is dominated by (2d). The term "large enough" for M_{ij} is ambiguous, and indeed the design of an effective M_{ij} is a challenging task that will be studied below.

Definition 1. *For every $(i,j)\in\mathcal{S}$, it is said that M_{ij} is feasible for the OTS if it preserves the equivalence between* (3) *and* (2f) *in the OTS. The smallest feasible M_{ij} is denoted by M_{ij}^{opt}.*

Remark 1. Note that the value of M_{ij}^{opt} is *independent* of the values of M_{rl}, for $(r,l)\in\mathcal{S}\backslash(i,j)$, in the linearized OTS formulation, as long as they are chosen to be feasible. In other words, given an instance of the OTS, the value of M_{ij}^{opt} is the same if M_{rl} satisfies $M_{rl}\ge M_{rl}^{\text{opt}}$ for every $(r,l)\in\mathcal{S}\backslash(i,j)$.

The problem under investigation in this section is the following: given an instance of OTS, is there an efficient algorithm to compute M_{ij}^{opt} or a good approximation of that for every $(i,j)\in\mathcal{S}$? It is desirable to find the smallest feasible values for every $M_{ij}, (i,j)\in\mathcal{S}$, in (3) because of two reasons:

1. Commonly used methods for solving MILP or MIQP problems, such as cutting-plane and branch-and-bound algorithms, are based on iterative convex relaxations of the constraints. Therefore, while a sufficiently large value for M_{ij} does not change the feasible region of the OTS after replacing (2f) with (3), it may have a significant impact on the feasible region of its convex relaxation. Small values for M_{ij} yield stronger convex relaxations with smaller feasible sets.
2. Large values for M_{ij} may cause numerical issues for convex relaxation solvers.

For every $(i,j)\in\mathcal{S}$, define $\mathcal{F}_{ij}$ as the set of all points $\{\mathbf{f},\mathbf{x},\mathbf{\Theta},\mathbf{p}\}\in\mathcal{F}$ such that $x_{ij}=0$.

Lemma 1. *The equation*

$$M_{ij}^{\text{opt}} = B_{ij} \times \max_{\{\mathbf{f},\mathbf{x},\mathbf{\Theta},\mathbf{p}\}\in\mathcal{F}_{ij}} \{|\theta_i-\theta_j|\} \tag{4}$$

holds for every flexible line $(i,j)\in\mathcal{S}$.

Proof. Consider a number M_{ij} such that $M_{ij}\ge B_{ij}\times \max_{\mathcal{F}_{ij}}\{|\theta_i-\theta_j|\}$. Every set $\{\mathbf{f},\mathbf{x},\mathbf{\Theta},\mathbf{p}\}\in\mathcal{F}$ satisfies (3) with the chosen M_{ij} and, hence, M_{ij} is feasible. Now, assume that $M_{ij} < B_{ij}\times\max_{\mathcal{F}_{ij}}\{|\theta_i-\theta_j|\}$. Based on the definition

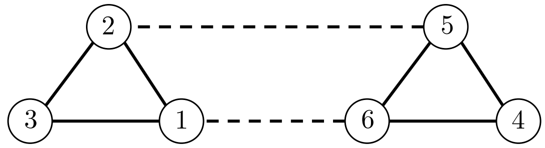


Fig. 1: The topology of the network in Example 1. The solid and dashed edges denote the lines with ON and OFF switches, respectively.

of the set $\mathcal{F}_{ij}$, this implies that there exists $\{\bar{\mathbf{f}}, \bar{\mathbf{x}}, \bar{\mathbf{\Theta}}, \bar{\mathbf{p}}\} \in \mathcal{F}$ such that $\bar{x}_{ij} = 0$, $\bar{f}_{ij} = 0$, and $M_{ij} < B_{ij}|\bar{\theta}_i - \bar{\theta}_j|$. Without loss of generality, suppose that $\bar{\theta}_i \geq \bar{\theta}_j$. Therefore, one can verify that

$$0 < B_{ij}(\bar{\theta}_i - \bar{\theta}_j) - M_{ij}(1 - \bar{x}_{ij}) \tag{5}$$

Combining with (3), this results in $\bar{f}_{ij} > 0$, contradicting the assumption $\bar{f}_{ij} = 0$. This completes the proof. □

Due to Lemma 1, the problem of finding M_{ij}^{opt} for every $(i,j) \in \mathcal{S}$ reduces to finding the $\max_{\mathcal{F}_{ij}}\{|\theta_i - \theta_j|\}$.

Remark 2. Note that, for a given $(i,j) \in \mathcal{S}$, the term $\max_{\mathcal{F}_{ij}}\{|\theta_i - \theta_j|\}$ is finite if and only if the buses i and j are connected for every feasible point in $\mathcal{F}_{ij}$. This means that the linearization of the OTS is well-defined if and only if the power network remains connected at every feasible solution in $\mathcal{F}_{ij}$ for all $(i,j) \in \mathcal{S}$.

The next example illustrates a scenario where the $\max_{\mathcal{F}_{ij}}\{|\theta_i - \theta_j|\}$ is not finite.

Example 1. Consider the network with 6 buses and 8 lines in Figure 1. Assume that the network is decomposed into two disjoint components (known as islands) with the buses $\{1,2,3\}$ and $\{4,5,6\}$ at a feasible point $\{\mathbf{f}, \mathbf{x}, \mathbf{\Theta}, \mathbf{p}\} \in \mathcal{F}_{16}$. Define $\tilde{\mathbf{\Theta}}$ as $\tilde{\theta}_i = \theta_i$ for $i \in \{1,2,3\}$ and $\tilde{\theta}_i = \theta_i + \tau$ for $i \in \{4,5,6\}$, where τ is an arbitrary scalar. It can be verified that $\{\mathbf{f}, \mathbf{x}, \tilde{\mathbf{\Theta}}, \mathbf{p}\} \in \mathcal{F}_{16}$ for every τ. Furthermore, $\tilde{\theta}_6 - \tilde{\theta}_1 = \theta_6 - \theta_1 + \tau$, which implies that $\max_{\mathcal{F}_{16}}\{|\tilde{\theta}_6 - \tilde{\theta}_1|\} \to +\infty$ as $\tau \to +\infty$.

To avoid unbounded values for M_{ij}^{opt}, the existence of a connected spanning subnetwork connecting all the nodes in the network with fixed lines will be assumed in the next section. In what follows, it will be shown that, even if $\max_{\mathcal{F}_{ij}}\{|\theta_i - \theta_j|\}$ is bounded for every $(i,j) \in \mathcal{S}$, one cannot devise an algorithm that efficiently finds $\max_{\mathcal{F}_{ij}}\{|\theta_i - \theta_j|\}$ since it amounts to an NP-hard problem. Furthermore, the impossibility of any constant factor approximation of $\max_{\mathcal{F}_{ij}}\{|\theta_i - \theta_j|\}$ in the linearized OTS is proven.

Theorem 1. *Consider an instance of the OTS together with a flexible line $(i,j) \in \mathcal{S}$, where $f_{kl}^{\max}$ is a given positive number for every $(k,l) \in \mathcal{L}\backslash\mathcal{S}$. Unless $P = NP$, it holds that:*

- *(Strong NP-hardness) there is no polynomial-time algorithm for finding $\max_{\mathcal{F}_{ij}}\{|\theta_i - \theta_j|\}$;*
- *(Inapproximability) there is no polynomial-time constant-factor approximation algorithm for finding $\max_{\mathcal{F}_{ij}}\{|\theta_i - \theta_j|\}$.*

Proof. To prove the strong NP-hardness of the problem, it suffices to show that there exists a polynomial reduction from the *longest path* problem in unweighted graphs–a well-known strongly NP-hard problem [34]. The longest path problem is defined as follows: Given an undirected graph $G(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ and $\mathcal{E}$ stand for the sets of vertices and edges, respectively, what is the longest simple path between two particular vertices i and j in $\mathcal{V}$? Let the length of the longest path be denoted as p^{opt}. We construct an instance of the OTS in the following way: Consider $|\mathcal{V}|$ buses and, for every $(r,l) \in \mathcal{E}$, connect buses r and l through a line with an arbitrary orientation that is equipped with a switch (note that $\mathcal{S} = \mathcal{E}$ in this case). For each line $(r,l) \in \mathcal{E}$, its susceptance and flow capacity are set to 1. For every bus $s \notin \{i,j\}$ in the system, we set $d_s = p_s^{\min} = p_s^{\max} = 0$, which implies that there is no load or generator connected to bus s. Connect a generator with $p_i^{\min} = p_i^{\max} = 1$ to bus i. Furthermore, connect a load $d_j = 1$ to bus j. Finally, set $r = 0$.

The instance designed above is feasible if and only if there is a simple path between buses i and j in G. Furthermore, the size of the constructed instance of the OTS is polynomial in the size of the instance of the longest path problem. Denote the feasible region of the designed instance of the OTS as $\mathcal{F}$. Note that $M_{ij}^{\text{opt}} = \max_{\mathcal{F}_{ij}}\{|\theta_i - \theta_j|\}$ due to Lemma (1). Without loss of generality, we drop the absolute value in the remainder of the proof. According to the defined characteristics of the loads and generators in the system, for any feasible solution of the OTS, there should be at least one simple path from bus i to bus j consisting of only lines that are switched on. Therefore, for every $(\mathbf{f}^*, \Theta^*, \mathbf{x}^*, \mathbf{p}^*) \in \arg\max_{\mathcal{F}_{ij}}\{\theta_i - \theta_j\}$, there exists a path $\mathcal{P}^* = \{(i, v_1), (v_1, v_2), ..., (v_k, j)\}$ with $x_{rk}^* = 1$ for all $(r,k) \in \mathcal{P}^*$. This simple path is visualized in Figure 2. With no loss of generality, assume that the direction of the flow on the lines respect the directions in $\mathcal{P}^*$. Based on Figure 2, one can verify that

$$\theta_i^* - \theta_j^* = \sum_{(r,l)\in\mathcal{P}^*} (\theta_r^* - \theta_l^*) = \sum_{(r,l)\in\mathcal{P}^*} f_{rl}^* \leq \sum_{(r,l)\in\mathcal{P}^*} f_{rl}^{\max} \leq p^{\text{opt}} \tag{6}$$

Now, it is desirable to construct a feasible solution $(\bar{\mathbf{f}}, \bar{\Theta}, \bar{\mathbf{x}}, \bar{\mathbf{p}}) \in \mathcal{F}$ that includes a simple path with lines that are switched on from buses i to j whose length is p^{opt}. To this end, consider the instance of the longest path problem and suppose that $\mathcal{P}^{\text{opt}} = \{(i, u_1), (u_1, u_2), ..., (u_l, j)\}$ defines the longest simple path in G between nodes i and j. For every flexible line (i,j) in the corresponding instance of the OTS, we set $\bar{x}_{ij}$ to 1 if this line belongs to $\mathcal{P}^{\text{opt}}$ and set to 0 otherwise. Moreover, we set $\bar{\theta}_j$ to 0 and define $\bar{\theta}_k = p_{kj}^{\text{opt}}$ for every bus k in $\mathcal{P}^{\text{opt}}$, where p_{kj}^{opt} is the length of the unique path between buses k and j in $\mathcal{P}^{\text{opt}}$. This yields that $\bar{f}_{rl}$ is equal to 1 for every line (r,l) in $\mathcal{P}^{\text{opt}}$. Furthermore, for every flexible line (t,s) that does not belong to $\mathcal{P}^{\text{opt}}$, we set $\bar{f}_{ts}$ to 0. To satisfy (2h), set $\bar{p}_i = 1$. Therefore, a feasible solution $(\bar{\mathbf{f}}, \bar{\Theta}, \bar{\mathbf{x}}, \bar{\mathbf{p}})$ is constructed that satisfies the following property:

$$\theta_i^* - \theta_j^* \geq \bar{\theta}_i - \bar{\theta}_j = \bar{\theta}_i = p^{\text{opt}} \tag{7}$$

Inequality (7) together with (6) establishes the proof of the strong NP-hardness of finding $\max_{\mathcal{F}_{ij}}\{|\theta_i - \theta_j|\}$. The inapproximability of the problem follows from the fact that, unless $P = NP$, there is no polynomial-time constant-factor

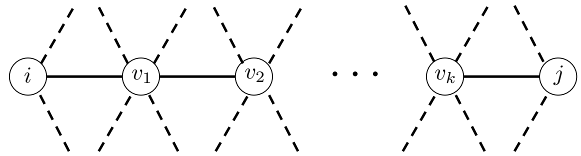


Fig. 2: The visualization of the path $\mathcal{P}^*$ in the proof of Theorem 1. The solid edges denote the lines in $\mathcal{P}^*$ (with ON switches) and the dashed edges correspond to the remaining lines.

approximation algorithm for determining the longest path between nodes i and j in G. □

Theorem 1 together with Lemma 1 implies that finding M_{ij}^{opt} is both strongly NP-hard and inapproximable within any constant factor, hence providing a negative answer to the question raised in this section.

Remark 3. The decision version of the OTS is known to be NP-complete [17]. One may speculate that the NP-hardness of finding the best M_{ij} for every $(i,j) \in \mathcal{S}$ may follow directly from that result. However, notice that there are some well-known problems with disjunctive constraints, such as the minimization of total tardiness on a single machine, which are known to be NP-hard [35] and yet there are efficient methods to find the optimal parameters of their big-M reformulation [36]. Theorem 1 shows that not only is finding the best M_{ij} for the OTS NP-hard, but one cannot hope for obtaining a strong linearized reformulation of the problem based on the big-M method.

Note that one may choose to use McCormick inequalities [25] instead of the big-M method to obtain a linear reformulation of the bilinear constraint (2f). In what follows, it will be shown that the complexity of finding the optimal parameters of McCormick inequalities is the same as those in the big-M method for the OTS. The McCormick inequalities can be written in the following form for a flexible line (i,j):

$$f_{ij} \le u_{ij|x_{ij}=1} x_{ij}, \tag{8a}$$
$$f_{ij} \ge l_{ij|x_{ij}=1} x_{ij}, \tag{8b}$$
$$f_{ij} \le B_{ij}(\theta_i - \theta_j) - l_{ij|x_{ij}=0} x_{ij}, \tag{8c}$$
$$f_{ij} \ge B_{ij}(\theta_i - \theta_j) - u_{ij|x_{ij}=0} x_{ij}, \tag{8d}$$

where $u_{ij|x_{ij}=1}$ and $l_{ij|x_{ij}=1}$ are the respective upper and lower bounds for $B_{ij}(\theta_i - \theta_j)$ in the case where the line (i,j) is in service. Similarly, $u_{ij|x_{ij}=0}$ and $l_{ij|x_{ij}=0}$ are the respective upper and lower bounds for $B_{ij}(\theta_i - \theta_j)$ when the switch for the flexible line (i,j) is off. It can be verified that the following equalities hold:

$$u_{ij|x_{ij}=1} = f_{ij}^{\max}, \tag{9a}$$
$$l_{ij|x_{ij}=1} = -f_{ij}^{\max}, \tag{9b}$$
$$u_{ij|x_{ij}=0} = B_{ij} \times \max_{\mathcal{F}_{ij}} \{\theta_i - \theta_j\}, \tag{9c}$$
$$l_{ij|x_{ij}=0} = B_{ij} \times \min_{\mathcal{F}_{ij}} \{\theta_i - \theta_j\}. \tag{9d}$$

Therefore, Theorem 1 immediately results in the NP-hardness and inapproximability of the pair $(l_{ij|x_{ij}=0}, u_{ij|x_{ij}=0})$.

IV. OTS WITH A FIXED CONNECTED SPANNING SUBGRAPH

In this section, we consider a power system with the property that the set of fixed lines contains a connected spanning tree of the power system. The objective is to show that a non-trivial upper bound on M_{ij} can be efficiently derived by solving a shortest path problem. Furthermore, it will be proven that this upper bound is tight in the sense that there exist instances of the OTS with a fixed connected spanning subgraph for which this upper bound equals M_{ij}^{opt}. Before presenting this result, it is desirable to state that the OTS is hard to solve even under the assumption of a fixed connected spanning subgraph.

Theorem 2. *The OTS with a fixed connected spanning subgraph is NP-hard.*

Proof. The proof is based on a reduction from subset sum problem [34] and a slight modification of the argument made in the proof of Theorem 3.1 in [17]. The details can be found in the Appendix. □

Remark 4. Unlike Theorem 1, the statement of Theorem 2 does not imply the strong NP-hardness of the OTS problem with a fixed connected spanning subgraph since the subset sum problem is only weakly NP-hard. Instead, it implies that this problem may be efficiently solvable if the capacity and the susceptance of the lines are small. However, note that small upper bounds on the angle difference between two neighboring buses does not directly translate into small line capacities. To illustrate, assume that $|\theta_i - \theta_j|$ is upper bounded by 25 degrees (≈ 0.43 radians) for a fixed line (i,j), which means that the capacity of this line is equal to $0.43 B_{ij}$. Therefore, despite having a small value for the angle difference, a large susceptance will lead to a large capacity, thereby rendering the OTS problem difficult to solve. Indeed, we have observed for Polish systems that the susceptance of some lines can be as large as $16,667$ per unit, which clearly cancels the positive effect of small angle differences.

Consider a feasible point $\{\mathbf{f}, \mathbf{x}, \boldsymbol{\Theta}, \mathbf{p}\} \in \mathcal{F}$. For any line $(i,j) \in \mathcal{L}$, we have

$$B_{ij}(\theta_i - \theta_j) = B_{ij} \sum_{(r,l) \in \mathcal{P}_{ij}} (\theta_r - \theta_l), \tag{10}$$

where $\mathcal{P}_{ij}$ is an arbitrary path from node i to node j in the fixed spanning connected subgraph of G. Together with Lemma 1, this implies that

$$\begin{aligned} M_{ij}^{\text{opt}} &= B_{ij} \times \max_{\{\mathbf{f},\mathbf{x},\boldsymbol{\Theta},\mathbf{p}\} \in \mathcal{F}_{ij}} \{|\theta_i - \theta_j|\} \\ &= B_{ij} |\theta_i^{\text{opt}} - \theta_j^{\text{opt}}| \\ &\le B_{ij} \sum_{(r,l) \in \mathcal{P}_{ij}} |\theta_r^{\text{opt}} - \theta_l^{\text{opt}}| \\ &\le B_{ij} \sum_{(r,l) \in \mathcal{P}_{ij}} \frac{f_{rl}^{\max}}{B_{rl}}, \end{aligned} \tag{11}$$

where $\{\mathbf{f}^{\text{opt}}, \Theta^{\text{opt}}, \mathbf{x}^{\text{opt}}, \mathbf{p}^{\text{opt}}\} \in \arg\max_{\mathcal{F}_{ij}} \{|\theta_i - \theta_j|\}$. Note that (11) holds for every path $\mathcal{P}_{ij}$ in the fixed connected spanning subgraph of the network. We will use this observation in

Theorem 3 to derive strong upper bounds for M_{ij}^{opt}. Denote the undirected weighted subgraph induced by the fixed lines in the power system as $G_{\mathcal{I}}(\mathcal{B}_{\mathcal{I}}, \mathcal{W}_{\mathcal{I}})$, where $\mathcal{B}_{\mathcal{I}} = \mathcal{B}$ and $\mathcal{W}_{\mathcal{I}}$ is the set of all tuples (i, j, w_{ij}) such that $(i, j) \in \mathcal{L} \backslash \mathcal{S}$ and w_{ij} is the weight corresponding to (i, j) defined as $f_{ij}^{\max}/B_{ij}$. Let $\mathcal{P}_{\mathcal{I};ij}$ and $p_{\mathcal{I};ij}$ be the set of edges in a shortest simple path between nodes i and j in $G_{\mathcal{I}}$ and its length, i.e., the summation of the weights of the edges in $\mathcal{P}_{\mathcal{I};ij}$, respectively.

Theorem 3. *For every flexible line $(i, j) \in \mathcal{S}$, the inequality*

$$M_{ij}^{\text{opt}} \leq B_{ij} \times p_{\mathcal{I},ij} \tag{12}$$

holds. Moreover, there exists an instance of the OTS for which this inequality is tight.

Proof. Based on (11), we have

$$M_{ij}^{\text{opt}} \leq B_{ij} \sum_{(r,l) \in \mathcal{P}_{\mathcal{I},ij}} \frac{f_{rl}^{\max}}{B_{rl}} = B_{ij} \sum_{(r,l) \in \mathcal{P}_{\mathcal{I},ij}} w_{rl} = B_{ij} \times p_{\mathcal{I},ij}. \tag{13}$$

Furthermore, a simple 3-bus system can be designed to show the tightness of the derived upper bound: consider a 3-bus network with the buses labeled as 1, 2, and 3. Assume that the lines $(1, 2)$ and $(2, 3)$ are fixed and the line $(1, 3)$ is flexible. Furthermore, suppose that the capacity and the susceptance of all lines are equal to 1. Upon connecting a generator with unit capacity ($p_1^{\max} = 1$ and $p_1^{\min} = 0$) to node 1 and a unit load to node 3, one can easily certify that $M_{13}^{\text{opt}} = 2$ which in turn equals to

$$B_{13} \left(\frac{f_{12}^{\max}}{B_{12}} + \frac{f_{23}^{\max}}{B_{23}} \right) = 2, \tag{14}$$

thereby verifying the tightness of (12) for this instance. □

Theorem 3 proposes a bound strengthening scheme for every flexible line in the OTS that can be carried out as a simple preprocessing step before solving the OTS using any branch-and-bound method. The algorithm for the proposed bound strengthening method is described in Algorithm 1.

Data: $G_{\mathcal{I}}(\mathcal{B}_{\mathcal{I}}, \mathcal{W}_{\mathcal{I}})$ and $B = \{B_{ij} | (i, j) \in \mathcal{S}\}$
Result: M_{ij} for every $(i, j) \in \mathcal{S}$
for $(i, j) \in \mathcal{S}$ **do**
  find $p_{\mathcal{I};ij}$ using Dijkstra's algorithm;
  $M_{ij} \leftarrow B_{ij} \times p_{\mathcal{I};ij}$;
end

Algorithm 1: Bound strengthening method for linearized OTS with fixed connected spanning subgraph

The worst-case complexity of performing this preprocessing step is $O(n_s n_b^2)$ since it is equivalent to performing n_s rounds of Dijkstra's algorithm on the weighted graph $G_{\mathcal{I}}$ (it can also be reduced to $O(n_s(n_l - n_s + n_b \log n_b))$ if the algorithm is implemented using a Fibonacci heap) [34]. This preprocessing step can be processed in an offline fashion before realizing the demand in the system. The impact of this preprocessing step on the runtime of the solver will be demonstrated on different cases in Section V.

As mentioned in the Introduction, the existence of a fixed connected spanning subgraph in power systems is a practical assumption since power operators should guarantee the reliability of the system by ensuring the connectivity of the power network. Therefore, due to Theorem 3, one can design non-conservative values for M_{ij}'s in order to strengthen the convex relaxation of OTS.

Remark 5. In practice, the angle difference between a pair of buses is tightly constrained if they are connected via a line. In other words, $|\theta_i - \theta_j|$ is constrained to be small if the line (i, j) is in service. One may conjecture that this can directly result in small values for M_{ij}^{opt}. In what follows, we will provide an easy and intuitive counterexample. Consider a 101-bus power system whose buses are labeled as $1, 2, ..., 101$. Define the set of lines as $\mathcal{L} = \{(i, i+1) | i = 1, 2, ..., 100\} \cup (101, 1)$ (note that the lines form a cycle). Furthermore, assume that all lines are fixed except for the line $(101, 1)$. Suppose that the upper bound on the angle difference between every two neighboring buses is set to 10 degrees. This implies that $|\theta_{101} - \theta_1|$ can be as large as 1000 degrees (17 radians) if $x_{101,1} = 0$ at a feasible solution of the OTS. Assume that the susceptance of the lines $(i, i+1)$ is 100 p.u. for every $i = 1, 2, ..., 100$ and the susceptance of the line $(101, 1)$ is 50 p.u.. Lemma 1 implies that $M_{ij}^{opt} \approx 1700$. Now, assume that there is a load in the amount of 17 p.u. at bus 101 and that a generator with the capacity 17 is connected to bus 1. One can easily verify that there exists a single feasible solution for the OTS in this case (independent of the objective function). Furthermore, any value for M_{ij} smaller than 1700 will cut this feasible solution and, hence, make the linearized OTS infeasible.

Consider the cost function for the OTS. In practice, a quadratic objective function is often used for production planning in order to model the cost of production, especially for thermal generators [37]. However, the nonlinearity introduced by a quadratic cost function makes the OTS particularly hard to solve. The main challenge of solving the MIQP is the fact that the optimal solution of its continuous relaxation often lies in the interior or on the boundary of its relaxed feasible region which may be infeasible for the original MIQP (as opposed to the extreme point solutions in MILP). More precisely, even obtaining the convex hull of the feasible region is not enough to guarantee the exactness of such continuous relaxations, since the optimal solution of the relaxed problem usually does not correspond to an extreme point in the convex hull if the objective function is quadratic. This introduces fractional solutions for the binary variables of the problem in most of the iterations of branch-and-bound methods which often leads to a high number of iterations. One way to partially remedy this problem is to reformulate the problem by introducing auxiliary variables such that a new linear function is minimized and the old quadratic objective function is moved to the constraints. This guarantees that the continuous relaxation of the reformulated problem will obtain an optimal solution that is an extreme point of the relaxed feasible region. This is a key reason behind the success of different conic relaxation and strengthening methods in MIQP [38], [39].

Assume that the objective function is quadratic in the form $\sum_{i=1}^{n_g} g_i(p_i)$, where $g_i(p_i) = a_i \times p_i^2 + b_i \times p_i$. Upon defining a new set of variables t_i for $i \in \mathcal{G}$, one can reformulate the objective function as $\sum_{i=1}^{n_g} \tilde{g}_i(p_i, t_i)$ where

$$\tilde{g}_i(p_i, t_i) = a_i \times t_i + b_i \times p_i. \tag{15}$$

subject to the additional convex constraints

$$p_i^2 \leq t_i, \qquad \forall i \in \mathcal{G} \tag{16}$$

To streamline the presentation, this problem is referred to as *conic formulation* of OTS whereas the previous formulation with quadratic objective function is called *quadratic formulation* henceforth.

V. Numerical Results

In this section, numerical studies on different test cases are conducted to evaluate the effectiveness of the proposed preprocessing method in solving the OTS. To this goal, we compare the proposed bound strengthening method to two different approaches:

- **Conservative approach:** In this method, the underlying structure of the power system is not exploited and a conservative value is chosen for every M_{ij}.
- **Restrictive approach:** In this method, additional constraints are imposed on the absolute value of the angles at all buses in order to obtain a small upper bound for M_{ij}'s. This comes at the expense of a shrinkage in the feasible region of the OTS and, hence, carries the risk of eliminating the globally optimal solution.

In the conservative approach, M_{ij} is chosen as $B_{ij} \sum_{(k,l)\in\mathcal{L}} f_{kl}^{\max}/B_{kl}$ for every $(i,j) \in \mathcal{S}$. This conservative value does not exploit the underlying structure of the network. There is also another upper bound on M_{ij} that does not take advantage of the underlying connectivity of the network. To describe the construction of this upper bound, for a given power network with n_b buses and n_l lines, let $\mathcal{T}$ collect the numbers $f_{kl}^{\max}/B_{kl}$ for all $(k,l) \in \mathcal{L}$ and set M_{ij} as the sum of the $n_b - 1$ largest elements in $\mathcal{T}$ multiplied by B_{ij}. First, note that this quantity is greater than or equal to $B_{ij} \times p_{\mathcal{I},ij}$ and, therefore, is a valid upper bound on M_{ij}^{opt} according to Theorem 3. Second, this number is clearly less conservative than the value $B_{ij} \sum_{(i,j)\in\mathcal{L}} f_{ij}^{\max}/B_{ij}$. However, we have observed in simulations that there is no improvement in the runtime of the solver using these upper bounds compared to the chosen values $B_{ij} \sum_{(i,j)\in\mathcal{L}} f_{ij}^{\max}/B_{ij}$. A detailed analysis of the the effect of these two upper bounds on the runtime of the solver can be found in Appendix.

Many studies on OTS in the literature use a restrictive approach and consider an additional set of constraints on the absolute value of the angles in the form of $|\theta_i| \leq \theta_i^{\max}$ in order to circumvent the issue of large values for M_{ij}'s [3], [10], [11], [16]. Under this new set of constraints, M_{ij} is upper bounded by $B_{ij}(\theta_i^{\max} + \theta_j^{\max})$. This quantity can be small if upper bounds for the absolute values of the angles are chosen to be small. However, imposing these types of constraints has no physical or safety justifications. Indeed, the stability and accuracy of the DC approximation is guaranteed by imposing strict constraints on the angle differences as opposed to the individual angles.

All of the test cases are chosen from the publicly available MATPOWER package [40], [41]. The simulations are run on a laptop computer with an Intel Core i7 quad-core 2.50 GHz CPU and 16GB RAM. The results reported in this section are for a serial implementation in MATLAB using the CVX framework and the GUROBI 6.00 solver with the default settings. The relative optimality gap threshold is defined as

$$\frac{z_{UB} - z_{LB}}{z_{UB}} \times 100,$$

where z_{UB} and z_{LB} are the objective value corresponding to the best found feasible solution and the best found lower bound, respectively. If the solver obtains a feasible solution for the OTS with the relative optimality gap of at most 0.1% within a time limit (to be defined later), it is said that an optimal solution is found.

A. Data Generation

First, we study the IEEE 118-bus system. There are 185 lines in this test case. In all of the considered instances, a randomly generated connected spanning subgraph of the network with 120 fixed lines is chosen and the remaining lines are considered flexible. To generate multiple instances of the OTS, the loads are multiplied by a load factor α chosen from the set $\{\alpha_1, \alpha_2, ..., \alpha_k\}$. Furthermore, a uniform line rating is considered for all lines in the system. We examine both linear and quadratic cost functions and perform the following comparisons:

- For the instances with a linear cost function, the total runtime of the solver is computed for the conservative and proposed bound strengthening methods (denoted by `L-C` and `L-P`, respectively) for different load factors and cardinality lower bounds.
- For the instances with a quadratic cost function, the runtime is computed for four different formulations: 1) the conic formulation with the proposed bound strengthening method (denoted by `C-P`), 2) the conic formulation with conservative approach (denoted by `C-C`), 3) the quadratic formulation with the proposed bound strengthening method (denoted by `Q-P`), and 4) the quadratic formulation with conservative approach (denoted by `Q-C`).

We also study six different large-scale Polish networks that are equipped with hundreds of switches. For each test case, a single load factor is considered for the OTS with linear and quadratic cost functions and the effect of the proposed bound strengthening method on the runtime and the optimality degree of the obtained solution is investigated compared to both conservative and restrictive approaches. Similar to the IEEE 118-bus case, we fix a randomly chosen connected spanning subgraph of the network with fixed lines. Similar to the previous works [3], [10], [11], [16], an upper bound of 0.6 radians (35 degrees) is chosen for the absolute value of the angles at every bus in the restrictive approach.

B. IEEE 118-bus System

In this subsection, the OTS is studied for the IEEE 118-bus system with 65 switches. Two types of cost functions are considered for this system:

Linear cost function: Figure 3a shows the runtime with respect to the various load factors. For all of these experiments, the lower bound on the cardinality of the ON switches is set to 45, i.e. $r = 45$ in (2i). It can be observed that, for small values of the load factor, the OTS is relatively easy to solve with a linear cost function and the solver can easily find the optimal solution within a fraction of second with or without the bound strengthening method. On the other hand, as the load factor increases, the OTS becomes harder to solve and the proposed bound strengthening method has a significant impact on the runtime. In particular, when the load factor equals 0.8, the strengthened formulation of the OTS is solved 8.73 faster.

In the second experiment, the performance of the solver is evaluated as a function of the lower bound on the number of the ON switches. As pointed out in [17], the OTS becomes computationally hard to solve with a relatively large cardinality lower bound. This can be a counter-intuitive observation; as this lower bound increases, the set of feasible solutions shrinks. However, a smaller feasible region does not necessarily result in fewer and faster branch-and-cut iterations. In fact, there are a number of cardinality-constrained NP-hard problems, such as *k-coverage* [42] or *subset selection in linear regression* [43], that become easy (and even trivial) when the cardinality constraint is removed from the formulation. Roughly speaking, this means that these types of constraints may shrink the feasible region, but instead can make the enumeration process harder. This becomes more evident by noting that one of earliest results on the NP-hardness of the OTS assumes a cardinality constraint on the number of switches [44]. This behavior is observed in Figure 3b. However, note that the negative effect of increased lower bound diminishes when the bound strengthening step is performed. Specifically, the strengthened formulation is solved 2.66 times faster on average for the first two cardinality lower bounds (10 and 20) and 6.53 times faster on average for the last two cardinality lower bounds (40 and 50).

Quadratic cost function: When the cost function is quadratic, the runtime of the solver is drastically increased. Nevertheless, the modified formulation of the OTS combined with the proposed bound strengthening method reduces the runtime significantly. For all experiments, a time limit of $3,000$ seconds is imposed. For those instances that are not solved within the time limit, the relative optimality gap that is achieved by the solver at termination is reported. The runtime for different formulations of the OTS with respect to various load factors is depicted in Figure 4a. Similar to the previous case, the lower bound on the cardinality of the switches is set to 45 for different load factors. It can be observed that when the load factor equals 0.5, the solver can find the optimal solution within the time limit only for `Q-ET`. As the load factor increases, the average runtime decreases for all formulations. As it is clear from Figure 4a, `Q-ET` significantly outperforms other formulations for all load factors. Specifically, the runtime for `Q-ET` is at least 5.95, 2.96, and 13.58 times faster than `Q-OT`, `Q-EC`, and `Q-OC` on average, respectively. Notice that these values are the under-estimators of the actual speedups since the solver was terminated before finding the optimal solution in many cases.

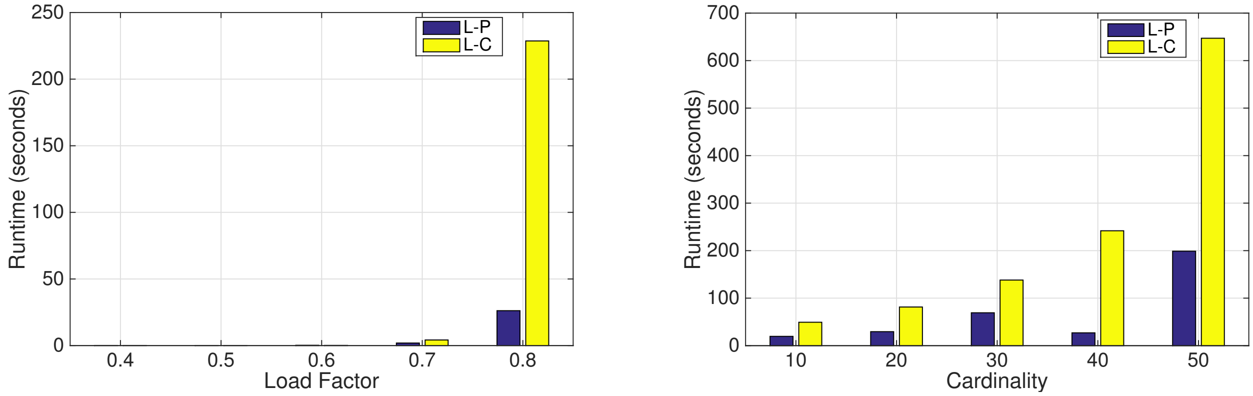
Next, consider the runtime for different formulations with respect to the change in the cardinality lower bound of ON switches. It can be observed in Figure 4b that the solution times for `Q-OT`, `Q-EC`, and `Q-OC` increase as the lower bound increases. This observation supports the argument made in [17] suggesting that a large lower bound on the cardinality of the ON switches would make the OTS harder to solve in general. However, notice that the cardinality constraint has a minor effect on the runtime of `Q-ET`. Notice that `Q-OC` has the worst runtime on average among different settings of the load factor and cardinality lower bound. This implies that the proposed reformulation of the objective function together with the bound strengthening step is crucial to efficiently solve the OTS with a quadratic objective function.

C. Polish Networks

In this part, the proposed bound strengthening method is applied to solve the OTS for Polish networks. As for the 118-bus system, the runtime is evaluated for both linear and quadratic cost functions. In all of the simulations, the cardinality lower bound on the number of ON switches is set to 0. The number of flexible lines varies from 70 to 400. The time limit is chosen as $14,400$ seconds (4 hours) for the solver. If the time limit is reached, the optimality gap of the best found feasible solution (if one exists) is reported. For the test cases with a quadratic cost function, only the modified formulation of the problem is considered because it significantly outperforms the original formulation.

Table I reports the performance and computational improvements when the bound strengthening method is incorporated into the formulation as a preprocessing step, compared to the conservative and restrictive approaches. This table includes the following columns:

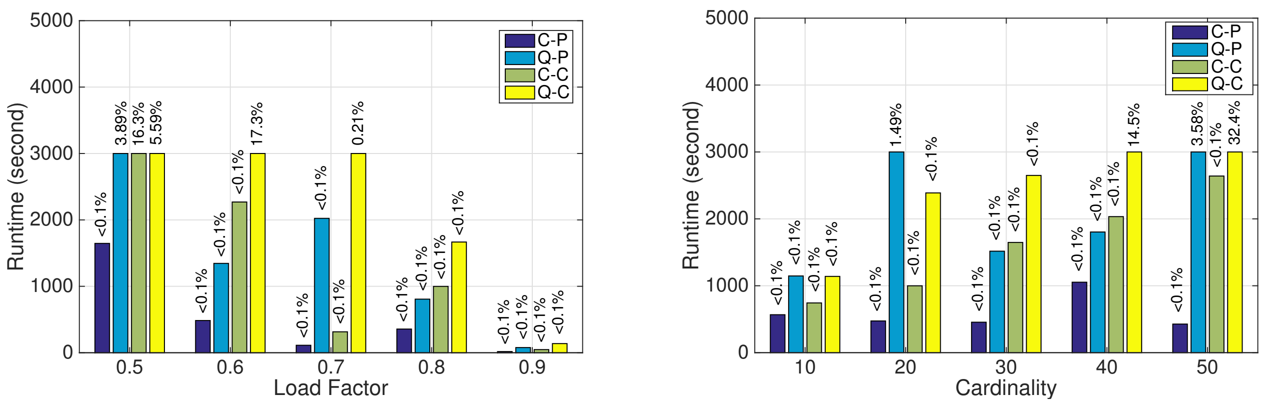
- Cost Function: The type of the cost function used in the simulation;
- # Cont.: The number of continuous variables in the system;
- # Binary: The number of binary variables corresponding to the flexible lines in the system;
- Time: The runtime (in seconds) for solving the OTS using different formulations within the time limit;
- Subopt: The sub-optimality of the derived solution using restrictive approach. This value quantifies the distance between the cost obtained using the restrictive approach and the optimal value of the cost function found via the proposed bound strengthening method. In particular, it is defined as
$$\frac{z_R - z_{BS}}{z_{BS}} \times 100 \tag{17}$$
where z_{R} and z_{BS} denote the optimal cost values of the restrictive and proposed methods, respectively. Note that the relative optimality gap threshold is still used to obtain the values of z_{R} and z_{BS};
- Pre. Time: The elapsed time of the proposed preprocessing step;
- Optgap: The relative optimality gap within the time limit. The solver is terminated when optgap is less than 0.1%;



(a) Runtime vs. load factor

(b) Runtime vs. cardinality lower bound

Fig. 3: The runtime of different formulations of OTS with a linear cost function with respect to different load factors and cardinality lower bounds. `L-C` and `L-P` correspond to the conservative and proposed bound strengthening methods, respectively.



(a) Runtime vs. load factor

(b) Runtime vs. cardinality lower bound

Fig. 4: The runtime of different formulations of OTS with a quadratic cost function with respect to different load factors and cardinality lower bounds. `C-P`, `C-C`, `Q-P`, and `Q-C` correspond to the conic formulation with the proposed bound strengthening method, the conic formulation with conservative approach, the quadratic formulation with the proposed bound strengthening method, and the quadratic formulation with conservative approach, respectively.

TABLE I: The performance of the solver with the proposed, conservative, and restrictive methods for Polish networks. The superscript * corresponds to the cases where the solver is terminated before finding the optimal solution due to the time limit.

				Restrictive		Proposed Method			Conservative		
Cases	Cost Function	# Cont.	# Binary	Time	Subopt	Pre. Time	Time	Optgap	Time	Optgap	Speedup
3120sp	Linear	3466	70	20	1.64%	<1	477	$<0.1\%$	3,623	$<0.1\%$	7.60
	Quadratic	3466	70	90	1.25%	<1	2,900	$<0.1\%$	14,400	0.12%	4.97*
2383wp	Linear	2789	80	14	0.76%	<1	418	$<0.1\%$	931	$<0.1\%$	2.23
	Quadratic	2789	80	44	0.52%	<1	252	$<0.1\%$	3,960	$<0.1\%$	15.71
2736sp	Linear	3105	100	11	0.95%	<1	80	$<0.1\%$	188	$<0.1\%$	2.35
	Quadratic	3105	100	15	1.14%	<1	156	$<0.1\%$	2,381	$<0.1\%$	15.26
3012wp	Linear	3516	120	50	0.49%	1	2,447	$<0.1\%$	14,400	0.11%	5.88*
	Quadratic	3516	120	162	0.23%	1	2,570	$<0.1\%$	14,400	0.11%	5.60*
3375wp	Linear	4053	200	320	1.33%	<1	98	$<0.1\%$	77	$<0.1\%$	0.79
	Quadratic	4053	200	490	2.70%	<1	4,301	$<0.1\%$	14,400	–	–
2746wop	Linear	3576	400	3,045	$<0.1\%$	1	17	$<0.1\%$	118	$<0.1\%$	6.94
	Quadratic	3576	400	7,238	$<0.1\%$	1	182	$<0.1\%$	3,523	$<0.1\%$	19.36
Average				**958**	**0.93%**	$\mathbf{<1}$	**1,158**	$\mathbf{<0.1\%}$	**6,033**	**0.1%***	**7.88***

- Speedup: The speedup in the runtime when the proposed bound strengthening method is used as a preprocessing step compared to the conservative approach.

It can be observed from Table I that the presented bound strengthening method can notably reduce the computation time compared to the conservative approach at no additional computational cost. In particular, the solver can be up to 19.36 times faster if the bound strengthening method is used to strengthen the formulation. Moreover, on average (excluding the case 3375wp with a quadratic cost function), the solution time is at least 7.88 times faster if the bound strengthening method is performed prior to solving the problem. For the case 3375wp with a quadratic cost function, the solver cannot obtain a feasible solution in $14,400$ seconds without bound strengthening. However, the solver can find an optimal solution within $4,301$ seconds after performing the proposed preprocessing step. The simplicity of the bound strengthening step is evident by the fact that this preprocessing step is carried out in less than 1 second in all of the experiments.

Furthermore, the solver cannot find a globally optimal solution of the OTS in most cases using the restrictive approach, due to the constraints imposed on the absolute values of the angles at all buses. In particular, the restrictive approach can increase the cost of the system operation by up to 2.70%. Furthermore, the proposed strengthened formulation results in 1% cost reduction on average, compared to the conventional restrictive approach. The runtime with the restrictive formulation is 17% less than the proposed method; however, the restrictive approach can only recover sub-optimal solutions for the OTS. In fact, the average runtime of the solver to obtain a solution with 1% (as opposed to 0.1%) relative optimality gap is only 508 seconds using the strengthened formulation.

To further elaborate on the effectiveness of the proposed strengthened formulation over the commonly used restrictive approaches, we study a modified version of the benchmark system 3375wp under different load scenarios. Similar to the previous case studies, a randomly chosen connected spanning subgraph of the network is fixed, and then 200 of the remaining lines are randomly selected and equipped with switches. We consider a linear objective for the generation cost, where the cost coefficient of each generator is chosen randomly from the interval $[20, 40]$. The load factors are chosen from the set $\{1, 1.05, 1.1, 1.15, 1.2\}$ and the line ratings are increased by 20% in order to guarantee the feasibility of the OTS for all load scenarios. It can be observed in Figure 5 that the runtime for the strengthened formulation is 66% less than that of the restrictive approach. Furthermore, it is evident that the restrictive approach results in sub-optimal solutions in all cases. In particular, the operational cost of the system with the load factor of 1.2 is increased by 6.48% when restrictive constraints are imposed on the absolute values of the angles at different buses. This clearly implies that the restrictive approach can significantly increase the operation cost in real-world networks and supports the premise of this work: *the proposed strengthened formulation strikes a good balance between the runtime of the solver and the objective of the derived solution*.

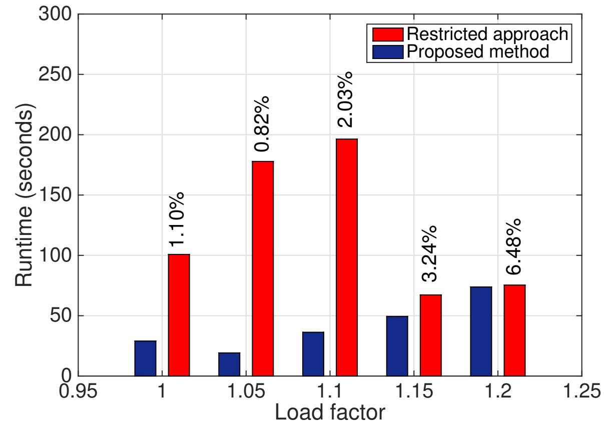


Fig. 5: The runtime of the restrictive and proposed formulations, together with the sub-optimality level of the restrictive approach, for the system case3375wp under different load factors.

VI. Conclusion

Finding an optimal topology of a power system subject to operational and security constraints is a daunting task. In this problem, certain lines are fixed/uncontrollable, whereas the remaining ones could be controlled via on/off switches. The objective is to co-optimize the topology of the grid and the parameters of the system (e.g., generator outputs). Common techniques for solving this problem are mostly based on mixed-integer linear or quadratic reformulations using the big-M or McCormick inequalities followed by iterative methods, such as branch-and-bound or cutting-plane algorithms. The performance of these methods partly relies on the strength of the convex relaxation of these reformulations. In this paper, it is shown that finding the optimal parameters of a linear or convex reformulation based on big-M or McCormick inequalities is NP-hard. Furthermore, the inapproximability of these parameters up to any constant factor is proven. Despite the negative results on the complexity of the problem, a simple bound strengthening method is developed to significantly strengthen mixed-integer reformulations of the OTS, provided that there exists a connected spanning subgraph of the network with fixed lines. This bound strengthening method can be used as a preprocessing step even in an offline fashion, before forecasting the demand in the system. Through extensive computational experiments, it is verified that this simple preprocessing technique can significantly improve the runtime of the mixed-integer solvers without sacrificing optimality as is done in standard formulations with restricting constraints in many test cases, including the IEEE 118-bus system and Polish networks.

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Appendix

A. Proof of Theorem 2

In this section, the proof of Theorem 2 is provided. We show that the decision version of the OTS with a fixed spanning subgraph, which is introduced below, is NP-complete:

Decision version of OTS (D-OTS): Given an instance of the OTS and a scalar C, is there a feasible solution for the OTS problem with the cost less than or equal to C?

To prove the NP-completeness of D-OTS, we adopt the approach in [17] and introduce a reduction from the subset sum problem that is a well-known NP-complete problem [34].

Subset sum problem: Given a set of non-negative integers a_i for $i = 1, 2, \ldots, n$ and a positive integer b, is there a subset $\mathcal{I} \in \{1, 2, \ldots, n\}$ such that $\sum_{i\in\mathcal{I}} a_i = b$?

Given an instance of the subset sum problem, we produce an instance of the D-OTS and show that the subset sum problem is feasible if and only if the designed instance of the D-OTS is feasible. Consider a network with $n + 3$ buses and $2n + 2$ lines constructed according to the following procedure:

1. For every $i = 1, 2, \ldots, n$, connect bus i to buses $n + 1$ and $n + 2$ via two lines with the capacity a_i/b and the susceptance $2a_i$. Furthermore, suppose that the line $(i, n + 1)$ is fixed and the line $(i, n + 2)$ is flexible for every $i = 1, 2, \ldots, n$.

TABLE II: Performance comparisons with two different conservative values for M_{ij}.

		Proposed Method		Conservative (1)			Conservative (2)		
Cases	Cost	Time	Optgap	Time	Optgap	Speedup	Time	Optgap	Speedup
3120sp	Linear	477	$< 0.1\%$	$3,623$	$< 0.1\%$	7.60	$4,586$	$< 0.1\%$	9.61
	Quadratic	$2,900$	$< 0.1\%$	$14,400$	0.12%	4.97*	$14,400$	0.13%	4.97*
2383wp	Linear	418	$< 0.1\%$	931	$< 0.1\%$	2.23	899	$< 0.1\%$	2.15
	Quadratic	252	$< 0.1\%$	$3,960$	$< 0.1\%$	15.71	$3,080$	$< 0.1\%$	12.22
2736sp	Linear	80	$< 0.1\%$	188	$< 0.1\%$	2.35	128	$< 0.1\%$	1.60
	Quadratic	156	$< 0.1\%$	$2,381$	$< 0.1\%$	15.26	$6,166$	$< 0.1\%$	39.52
3012wp	Linear	$2,447$	$< 0.1\%$	$14,400$	0.11%	5.88*	$14,400$	0.11%	5.88*
	Quadratic	$2,570$	$< 0.1\%$	$14,400$	0.11%	5.60*	$14,400$	0.11%	5.60*
3375wp	Linear	98	$< 0.1\%$	77	$< 0.1\%$	0.79	89	$< 0.1\%$	0.98
	Quadratic	$4,301$	$< 0.1\%$	$14,400$	–	–	$14,400$	0.15%	3.35*
2746wop	Linear	17	$< 0.1\%$	118	$< 0.1\%$	6.94	435	$< 0.1\%$	25.59
	Quadratic	182	$< 0.1\%$	$3,523$	$< 0.1\%$	19.36	316	$< 0.1\%$	1.74
Average		$\mathbf{1,158}$	$< \mathbf{0.1\%}$	$\mathbf{6,033}$	$\mathbf{0.1\%}^*$	$\mathbf{7.88}^*$	$\mathbf{6,108}$	$\mathbf{0.1\%}^*$	$\mathbf{9.43}^*$

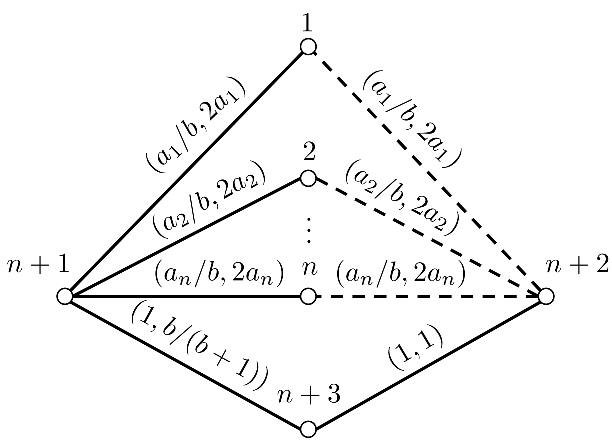

Fig. 6: The visualization of the instance of D-OTS designed in the proof of Theorem 2. The solid and dashed edges denote the fixed and flexible lines, respectively. The first and the second arguments of the tuple on every line denote its capacity and susceptance, respectively.

2. Connect bus $n+1$ to bus $n+3$ via a fixed line with capacity 1 and susceptance $b/(b+1)$.
3. Connect bus $n+2$ to bus $n+3$ via a fixed line with unit capacity and susceptance.

Figure 6 visualizes the constructed network. The cardinality lower bound r in (2i) is set to zero. A generator with capacity 2 is connected to bus $n+1$ and there is a load in the amount of 2 at bus $n+3$. Furthermore, assume that $g_{n+1}(p_{n+1})$ is zero. Finally, C (defined in the statement of D-OTS) is set to an arbitrarily chosen non-negative number. Based on this construction, the cost of every feasible solution for the OTS is zero. Therefore, addressing D-OTS reduces to verifying if the constructed instance of the OTS is feasible. First, we show that the feasibility of the subset sum problem implies the feasibility of the designed instance of the OTS. Consider a subset $\mathcal{I}$ such that $\sum_{i\in\mathcal{I}} a_i = b$. A feasible solution for the OTS is designed as follows:

- Set $\theta_{n+1} = 1 + \frac{1}{b}$, $\theta_{n+2} = 1$, $\theta_{n+3} = 0$, $\theta_i = 1 + \frac{1}{2b}$ for every $i \in \mathcal{I}$, and $\theta_i = 1 + \frac{1}{b}$ for every $i \notin \mathcal{I}$.
- Set $x_{i,n+2} = 1$ for every $i \in \mathcal{I}$ and $x_{i,n+2} = 0$ for every $i \notin \mathcal{I}$.

Based on the assigned values, one can easily verify that $p_{n+1} = 2$, $f_{n+1,n+3} = f_{n+2,n+3} = 1$, $f_{n+1,i} = f_{i,n+2} = \frac{a_i}{b}$ for every $i \in \mathcal{I}$, and $f_{n+1,i} = f_{i,n+2} = 0$ for every $i \notin \mathcal{I}$. Furthermore, all of the constraints in (2) are satisfied. This implies that the designed OTS is indeed feasible.

Next, suppose that OTS is feasible. Due to the assigned load at bus $n+3$ and the capacity of each line, we should have $f_{n+1,n+3} = f_{n+2,n+3} = 1$. Upon setting $\theta_{n+3} = 0$, one can verify that $\theta_{n+2} = 1$ and $\theta_{n+1} = 1 + \frac{1}{b}$. On the other hand, due to the power balance constraint (2h) at bus $n+2$, at least a flexible line should be in service. Denote the set of all flexible lines that are in service as $\mathcal{J}$. Given a bus i for which $(i, n+2) \in \mathcal{J}$, one can verify that $\theta_i = 1 + \frac{1}{2b}$. To show this, note that any value for θ_i other than $1 + \frac{1}{2b}$ violates the power balance constraint (2h) at bus i. This, together with (2h) at bus $n+2$, results in

$$\sum_{i\in\mathcal{J}} \frac{2a_i}{2b} = \sum_{i\in\mathcal{J}} \frac{a_i}{b} = 1 \tag{18}$$

which implies that the subset sum problem is feasible. This concludes the proof.

B. Comparison Between Conservative Bounds

In this section, we compare the runtime of the solver when different conservative bounds are used for M_{ij}'s in the big-M reformulation of the OTS. The results for Polish networks are summarized in Table II. In this table, Conservative (1) refers to the case where M_{ij}'s are chosen as $B_{ij}\sum_{(i,j)\in\mathcal{L}} f_{ij}^{\max}/B_{ij}$ and Conservative (2) corresponds to the case where the M_{ij} values are assigned according to the following procedure: for a given power network with n_b buses and n_l lines, let $\mathcal{T}$ collect the values of $f_{kl}^{\max}/B_{kl}$ for every line $(k,l) \in \mathcal{L}$ and set M_{ij} as the summation of the $n_b - 1$ largest elements in $\mathcal{T}$ multiplied by B_{ij}. It is observed in Table II that none of these conservative bounds can improve the runtime of the solver compared to the proposed strengthened bounds.

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